QUIZ 2 - CALCULUS 3 (2021/3/25)

Use the given table about the function f to answer problem 1 and problem 2.

Points	$f(x,y)$	$f_x(x,y)$	$f_y(x,y)$	$f_{xx}(x,y)$	$f_{xy}(x,y)$	$f_{yy}(x,y)$
A (1,5)	7	0	0	20	-3	0
B (2,-1)	0	0	7	9	6	4
C (0,3)	0	0	0	-12	10	-9
D (-1,0)	10	3	-4	-2	0	-4
E (-2,4)	4	0	0	4	5	6
F (-3,1)	-3	0	0	1	1	1
G (4,1)	-5	0	0	6	3	12

1. (4 pts) If $x(t) = \sin t - \cos t$ and $y(t) = \sin(2t)$, then at $t = 0$ it would pass through point D. Use the table and the Chain Rule to find $\left.\frac{df}{dt}\right|_{t=0}$.

   Solution:

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$
$$x'(0) = 1,\ y'(0) = 2 \quad \text{(1 pt each)}$$
$$\left.\frac{df}{dt}\right|_{t=0} = 3 \cdot 1 + (-4) \cdot 2 = 3 - 8 = -5 \quad \text{(2 pts)}$$

2. (5 pts) Determine which points given in the table are critical points of f. Use the Second Derivatives Test and state the conclusion for each critical point.

   Solution:

   Critical points: A (saddle), C (local max), E (saddle), F (inconclusive), and G (local min).

   (1 pt for each correct classification, -1 for each extra point in the list)

3. Let $F(x,y,z) = z^2\,(1 + \ln|xy|)$.

   (a) (3 pts) Find the gradient of F.

   (b) (2 pts) Find the directional derivative of F at the point $(1,1,3)$ in the direction of $\mathbf{u} = \frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$.

   (c) (3 pts) Find an equation of the tangent plane to the surface $F(x,y,z) = 9$ at the point $(1,1,3)$.

   Solution:

   (a) $\nabla F = \langle x^{-1}z^2, y^{-1}z^2, 2z(1 + \ln|xy|)\rangle$

   (b) $\nabla F(1,1,3) = \langle 9, 9, 6\rangle$

$$D_{\mathbf{u}}F(1,1,3) = \langle 9,9,6\rangle \cdot \mathbf{u} = 7$$

   (c) $9(x-1) + 9(y-1) + 6(z-3) = 0$

4. (3 pts) **List** the three equations you need to solve for using the Lagrange Multipliers method on finding the extreme values of $f(x,y) = ye^x$ with constraint $2x + y^3 = 24$. **Do not solve them!**

   Solution:

$$ye^x = 2\lambda, \quad e^x = 3y^2\lambda, \quad 2x + y^3 = 24$$